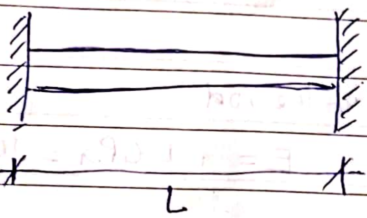
Temperature stress

Materials expand or contract with change in temperature. However if the material is restrained partialy or fully to resist the contraction / expansion, there will be a build up of stress in the material

Consider a body restrained at both ends



Let L = original lenght of bar
t = Change in temperature
α = Coefficient of linear expansion

the increase in lenght due to increase in temperature is given as

$$\delta l = L \times \alpha \times t$$

If expansion is prevented, then compressive strain is induced. It is given by

$$\varepsilon = \frac{\delta l}{L} = \frac{L \cdot \alpha \cdot t}{L} = \alpha t$$

Stress $\sigma = \varepsilon E$, $\varepsilon = \alpha t$

$$\sigma = \alpha \cdot t \cdot E$$

Example /

A rod is 2m long at a temperature of 10°C. Find the expansion of the rod when the temperature is raised to 80°C. If this expansion is prevented, find the stress induced in the rod as a result of the restraint

$E = 100\ GPa$, $\alpha = 0.000012/°C$

Soln

$l = 2m = 2\times10^3$ mm

$t = 80 - 10 = 70°C$

$\alpha = 0.000012/°C$

expansion in the rod $= \delta l = L \times \alpha \times t$

$\delta l = 2\times10^3 \times 0.000012 \times 70 = 1.68$ mm

Stress in the material of the rod

$\sigma = \alpha t E$, $E = 100$ GPa $= 100 \times 10^3$ N/mm^2

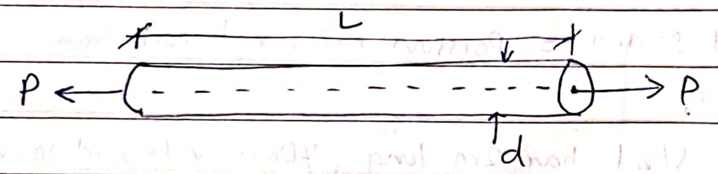
$\sigma = 0.000012 \times 70 \times 100 \times 10^3 = 84$ N/mm^2

$= \underline{\underline{84 \text{ MPa}}}$

Poisson Ratio

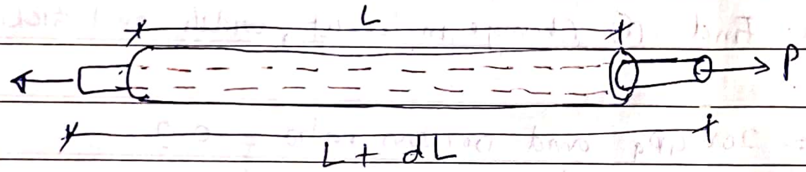
When an external force act on a body, it undergoes deformation

Consider a circular bar subjected to a tensile force as shown



L = lenght
d = diameter of bar

now deform to;



dL = change in lenght

when there is a tensile force (pull) on a material, the lenght increases but the cross-sectional area of the material reduces (reduction in diameter).

The deformation of the bar per unit lenght in the direction of the force is known as *

- primary or linear or longitudinal strain

Studying the deformation further we observed that

while L increase by dL, diameter reduced by δD

$\therefore$ in tensile stress we have $L + \delta L$ and $D - \delta D$

in compressive stress " $L - \delta L$ and $D + \delta D$

Therefore its obvious that every direct stress is accompanied by a strain in its own direction and an opposite kind of strain at right angle to it which is secondary or lateral strain

The ratio of lateral strain to longitudinal ~~longitudinal~~ (linear) strain is known as poisson ratio (μ)

$$\frac{\text{Secondary strain}}{\text{Primary strain}} = \frac{\text{lateral strain}}{\text{linear strain}} = \mu$$

i.e lateral strain = Poisson ratio x linear strain = $\mu\varepsilon$

Example / A steel bar 2m long, 40mm wide and 20mm thick is subjected to an axial pull of 160KN in the direction of its length. Find (i) Change in lengut, width and thickness of the bar.

$E = 200$ GPa and poisson ratio $= 0.3$

Soln

$L = 2m = 2 \times 10^3 mm$

width, $b = 40mm$, thickness $t = 20mm$,

$P = 160KN = 160 \times 10^3 N$, $E = 200 GPa = 200 \times 10^9 Pa$

$1 Pa = 1 N/m^2$, $\because 200 \times 10^9 Pa = 200 \times 10^9 N/m^2$

$\therefore E = 200 \times \frac{(1,000,0\cancel{00},0\cancel{00})}{1,0\cancel{00},0\cancel{00}} N/mm^2$

$= 200 \times 10^3 N/mm^2$

(i) Change in lengut δl

$$\delta l = \frac{PL}{AE} = \frac{(16\cancel{0} \times \cancel{10^3}) \times (\cancel{2} \times \cancel{10^3})}{(4\cancel{0} \times \cancel{20}) \times (2\cancel{00} \times \cancel{10^3})} = \frac{16}{8} = 2$$

$\delta l = 2mm$

linear strain ε = $\frac{dL}{L} = \frac{\cancel{2}}{\cancel{2} \times 10^3} = 0.001$

lateral strain = linear strain x poisson ratio = $\mu \varepsilon$

" $= 0.3 \times 0.001 = 0.0003$

(2) Change in width, δb = b x lateral strain

$= 40 \times 0.0003 = 0.012\,mm$

(3) Change in thickness δt = t x lateral strain

$= 20 \times 0.0003$

$= 0.006\,mm$

Example 2/

A metal bar 50mm X 50mm in section was subjected to an axial compression load of 500KN. If the contraction of the 200mm guage length was found to be 0.5mm and increase in thickness equal 0.04mm; Find

(i) Young modulus E

(ii) poisson ratio μ

Soln

$b = 50mm$, $t = 50mm$, $P = 500KN = 500 \times 10^3 N$, $L = 200mm$

$\delta L = 0.5mm$ and $\delta t = 0.04mm$

$dl = \frac{PL}{AE}$, $0.5 = \frac{(500 \times 10^3) \times 200}{50 \times 50 \times E}$

$E = \frac{500 \times 10^3 \times 200}{2500 \times 0.5} = 80 \times 10^3 N/mm^2$

or

lateral strain = $\mu \times$ linear strain

$$\frac{\text{lateral strain}}{\text{poisson ration}(\mu)} = \text{linear strain } (\delta l)$$

$$\text{poisson ratio } \mu = \frac{\text{lateral strain}}{\text{linear strain}}$$

$$\text{linear strain } \varepsilon = \frac{\delta l}{L} = \frac{0.5}{200} = 0.0025$$

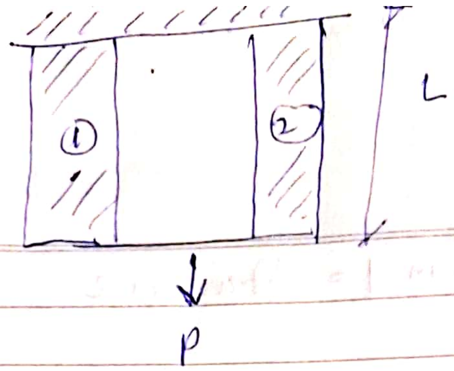
Change in thickness δt = thickness $\times$ lateral strain

$\delta t = 0.04mm = 50mm \times$ lateral strain

$$\text{lateral strain} = \frac{0.04}{50mm} = 0.0008$$

$$\text{Poisson ratio} = \frac{\text{lateral strain}}{\text{linear strain}} = \frac{0.0008}{0.0025} = 0.32$$

$$\underline{\underline{\mu = 0.32}}$$



A composite bar is made of two or more bars of equal lenght but different materials rigidly fixed with each other and behaving as one unit for extension or compression when subjected to either tension or compression forces.

$$P = P_1 + P_2$$

Stress in bar 1 = $\frac{\text{load carried by bar 1}}{\text{Area X-Section of bar 1}}$

$$\sigma_1 = \frac{P_1}{A_1} \quad \text{or} \quad P_1 = \sigma_1 A_1$$

Similarly $\sigma_2 = \frac{P_2}{A_2} = P_2 = \sigma_2 A_2$

Substituting $P = \sigma_1 A_1 + \sigma_2 A_2$

Strain in bar 1 = $\frac{\text{Stress in bar 1}}{\text{Young modulus of bar 1}} = \frac{\sigma_1}{E_1} = \varepsilon_1$

also $\varepsilon_2 = \frac{\sigma_2}{E_2}$

(6)

but Strain in 1 = Strain in 2

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\sigma_1 = \frac{E_1 \sigma_2}{E_2} \quad , \quad \frac{E_1}{E_2} = \text{modular ratio}$$